# Outline

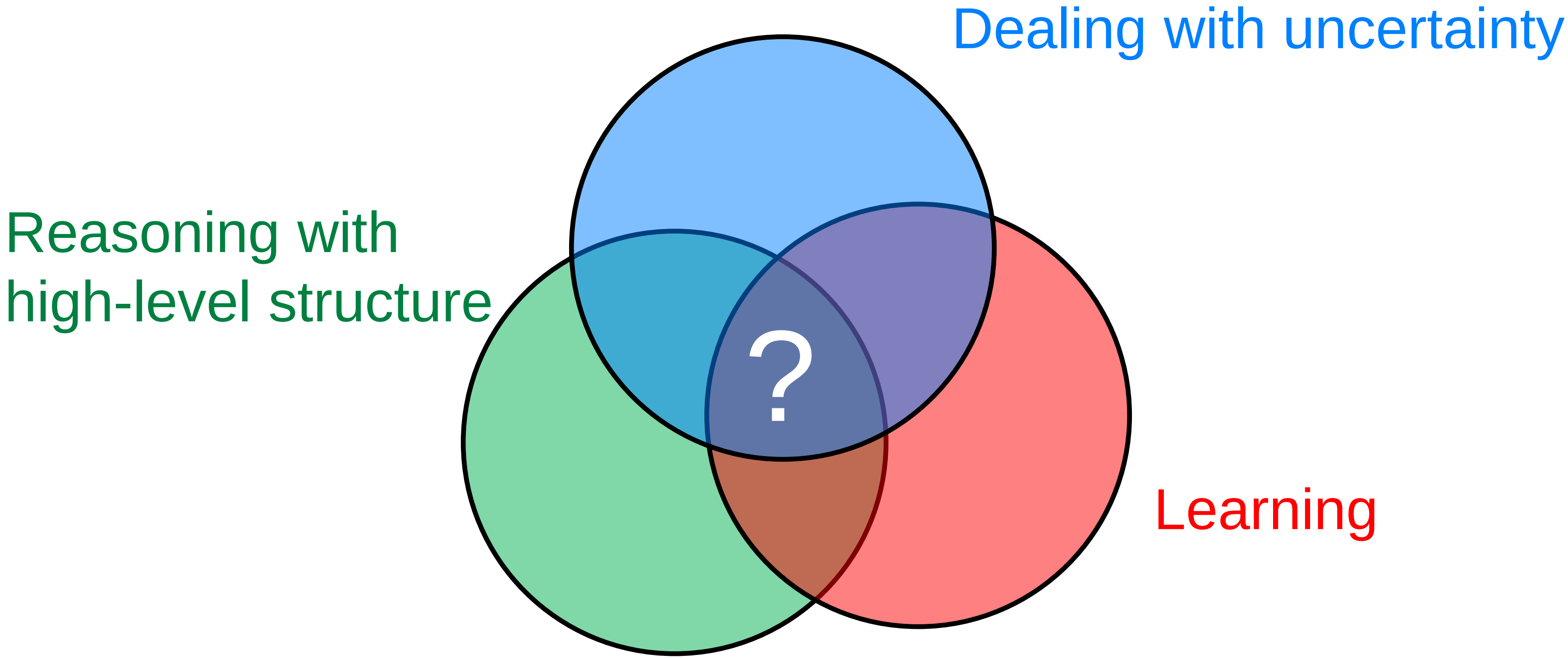
- Part 1: Motivation
- Part 2: Probabilistic Databases
- Part 3: Weighted Model Counting
- Part 4: Lifted Inference for WFOMC

- Part 5: Completeness of Lifted Inference
- Part 6: Query Compilation
- Part 7: Symmetric Lifted Inference Complexity
- Part 8: Open-World Probabilistic Databases
- Part 9: Discussion & Conclusions

# Summary

- Relational models = the vast majority of data today, plus probabilistic Databases
- Weighted Model Counting = Uniform approach to Probabilistic Inference
- Lifted Inference = really simple rules
- The Power of Lifted Inference = we can prove that lifted inference is better

# Challenges for the Future

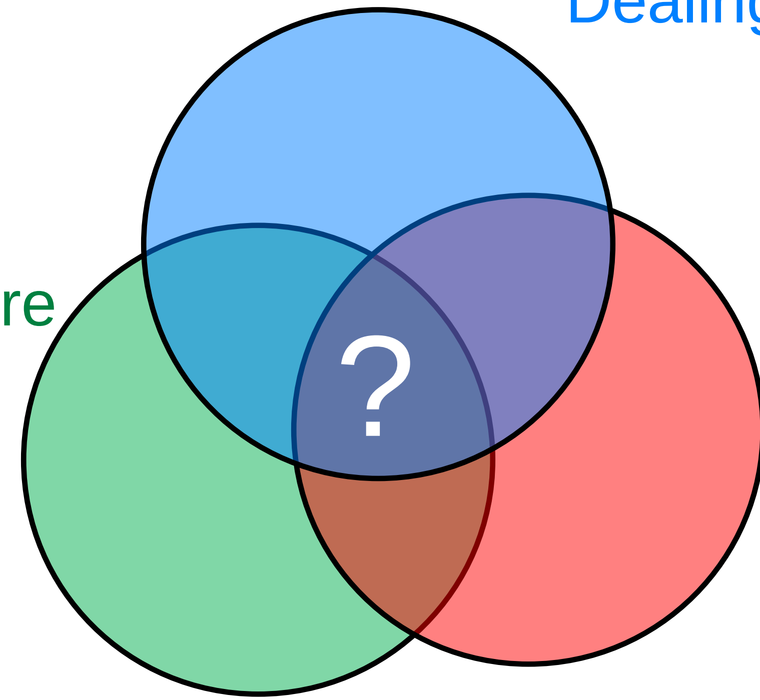


# Challenges for the Future

Dealing with uncertainty

Reasoning with
high-level structure

- logic
- databases
- programming
- ...




Learning

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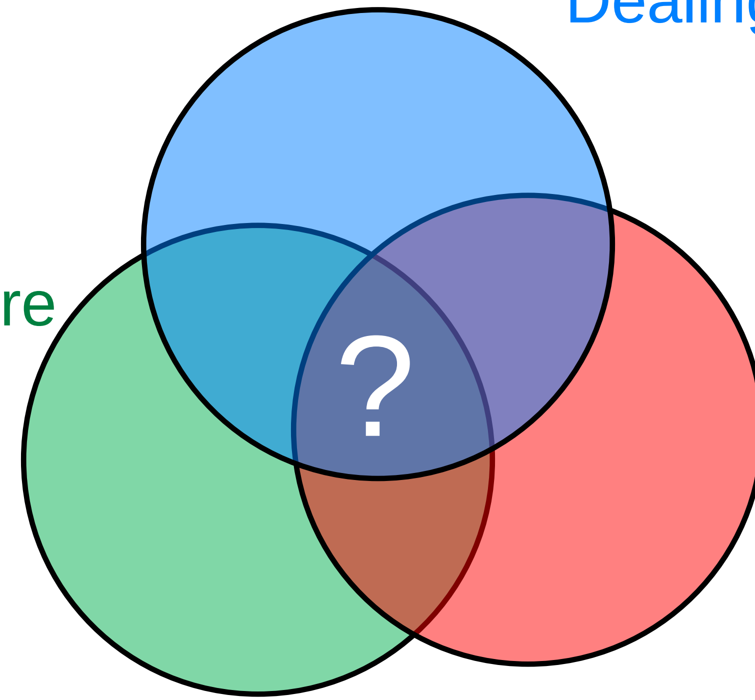


Learning

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Dealing with uncertainty

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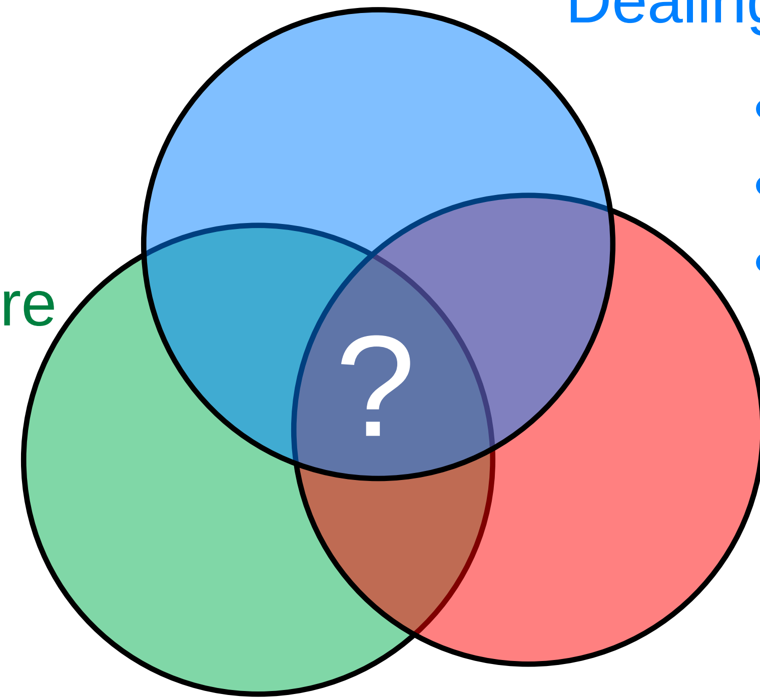


Learning

- parameters
- structure

# Challenges for the Future

Dealing with uncertainty

- probability theory
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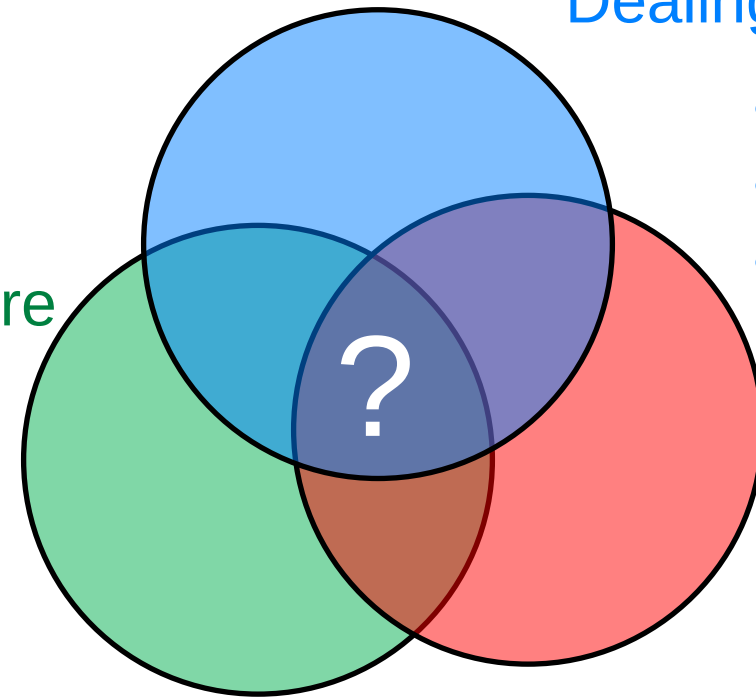



Learning

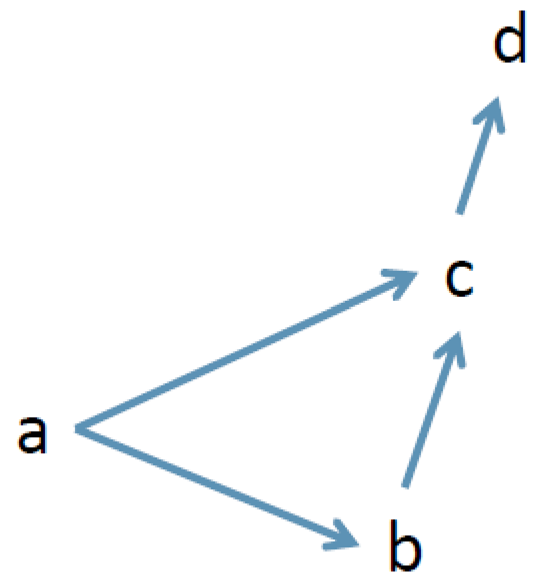
- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, probabilistic databases, ...

# Datalog

Edge

| x | y |
|---|---|
| a | c |
| a | b |
| b | c |
| c | d |



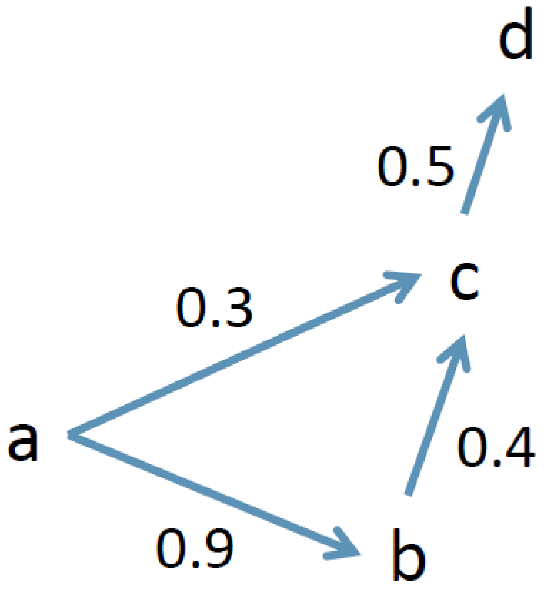

```
path(X,Y):- edge(X,Y).
path(X,Y):- edge(X,Z), path(Z,Y).
```

`path(a,d)` = Yes

# Probabilistic Datalog

Edge

| x | y | P |
|---|---|---|
| a | c | 0.3 |
| a | b | 0.9 |
| b | c | 0.4 |
| c | d | 0.5 |




```
path(X,Y) :- edge(X,Y).
path(X,Y) :- edge(X,Z), path(Z,Y).
```

P(**path(a,d)**) = ??

[De Raedt'07, Fierens'15]

# Probabilistic Programming

- Programming language + random variables
- Reason about distribution over executions

*As going from hardware circuits to programming languages*

```
sample(L,N,S) :- permutation(S,T), sample_ordered(L,N,T).

sample_ordered(_, 0, []).
sample_ordered([X|L], N, [X|S]) :-
     N > 0, sample_now([X|L],N), N2 is N-1,
     sample_ordered(L,N2,S).
sample_ordered([H|L], N, S) :-
     N > 0, \+ sample_now([H|L],N), sample_ordered(L,N,S).

P::sample_now(L,N) :- length(L, M), M >= N, P is N/M.
```

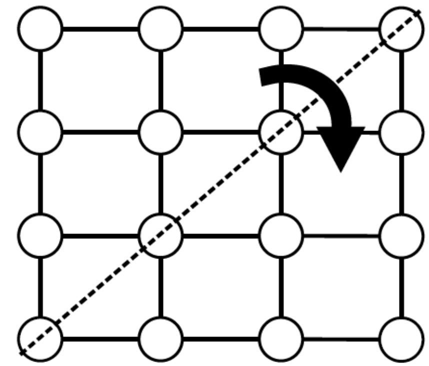
`P(sample([c,a,c,t,u,s],3,[c,a,t])) = 0.1`

[De Raedt'07, Fierens'15]

# Approximate Symmetries

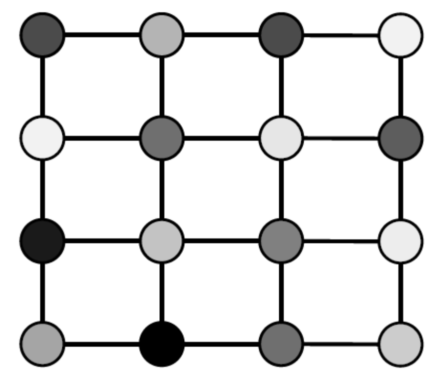
- What if not liftable? Asymmetric graph?
- Exploit approximate symmetries:
  - Exact symmetry g: $\Pr(\mathbf{x}) = \Pr(\mathbf{x}^g)$

    E.g. Ising model
    without external field



  - Approximate symmetry g: $\Pr(\mathbf{x}) \approx \Pr(\mathbf{x}^g)$

    E.g. Ising model with external field



P  ≈ P 

[VdB'13,'15,Gogate'14]

# Example: Statistical Relational Model

- WebKB: Classify pages given links and words
- Very large Markov logic network

$$1.3 \quad \texttt{Page}(\mathsf{x}, \mathsf{Faculty}) \Rightarrow \texttt{HasWord}(\mathsf{x}, \mathsf{Hours})$$

$$1.5 \quad \texttt{Page}(\mathsf{x}, \mathsf{Faculty}) \wedge \texttt{Link}(\mathsf{x}, \mathsf{y}) \Rightarrow \texttt{Page}(\mathsf{y}, \mathsf{Course})$$

and 5000 more …

- No symmetries with evidence on Link or Word
- Where do approx. symmetries come from?

[VdB'13,'15]

# Over-Symmetric Approximations

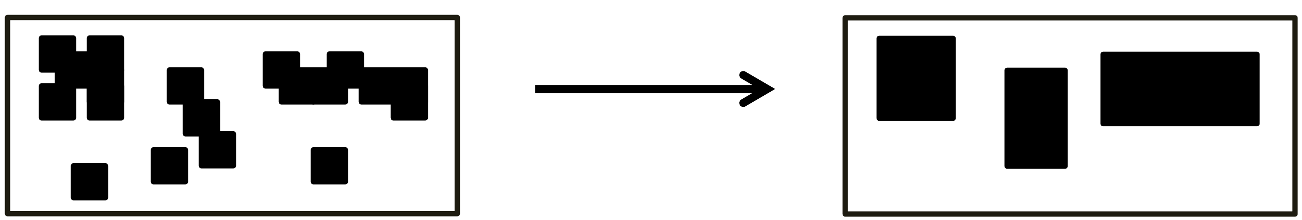
- OSA makes model more symmetric
- E.g., low-rank Boolean matrix factorization

Link (“aaai.org”, “google.com”)
Link (“google.com”, “aaai.org”)
Link (“google.com”, “gmail.com”)
Link (“ibm.com”, “aaai.org”)

→

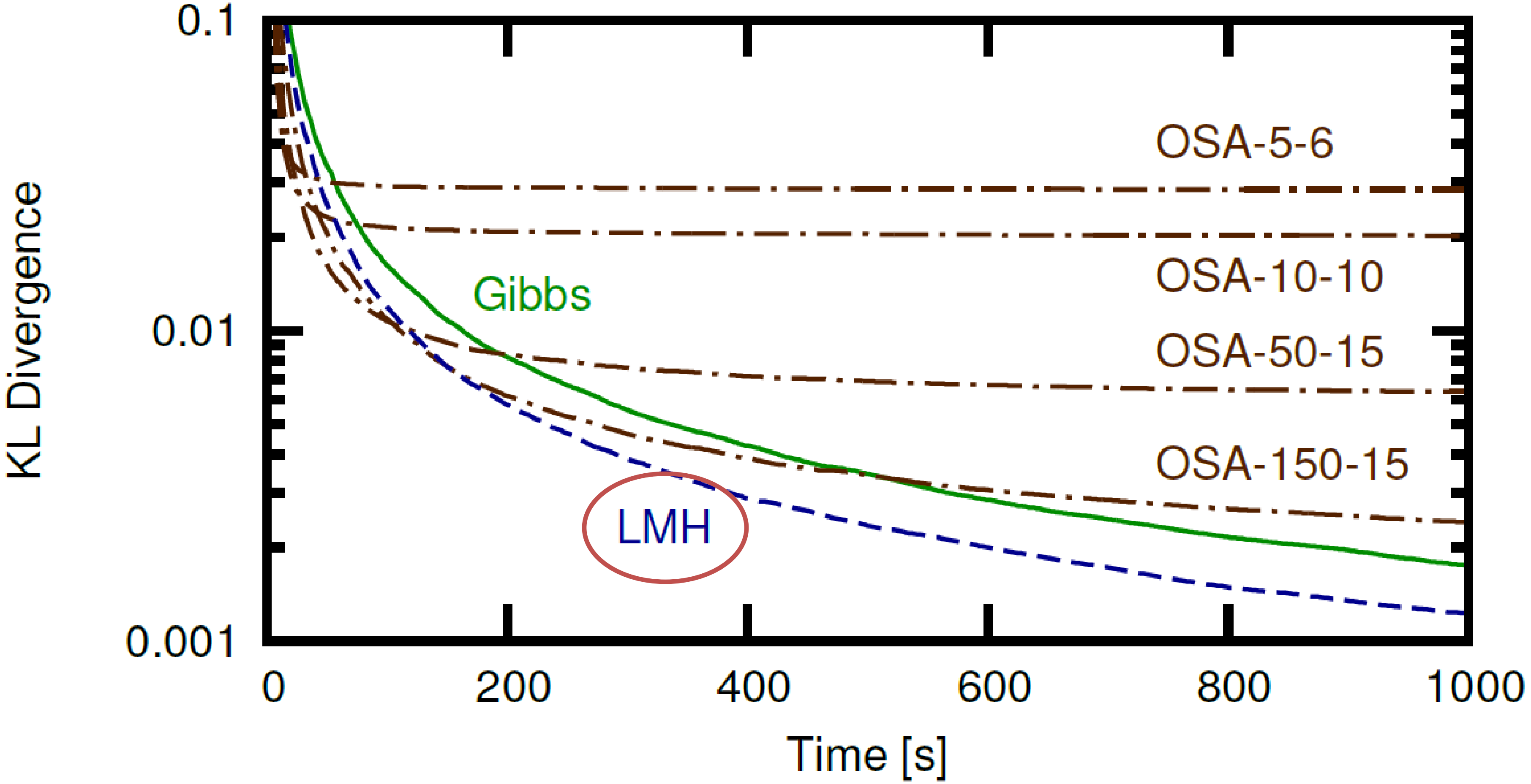
Link (“aaai.org”, “google.com”)
Link (“google.com”, “aaai.org”)
- ~~Link (“google.com”, “gmail.com”)~~
+ Link (“aaai.org”, “ibm.com”)
Link (“ibm.com”, “aaai.org”)

google.com and ibm.com become symmetric!



[VdB’13,’15]

# Experiments: WebKB




[VdB'13,'15]

# Lifted Weight Learning

- **Given:** A set of first-order logic **formulas**

  w FacultyPage(x) ∧ Linked(x,y) ⇒ CoursePage(y)

  A set of training **databases**
- **Learn:** The associated maximum-likelihood **weights**

$$\frac{\partial}{\partial w_j} \log \Pr_w(db) = n_j(db) - \mathbb{E}_w[n_j]$$

Count in databases
Efficient

Expected counts
Requires inference

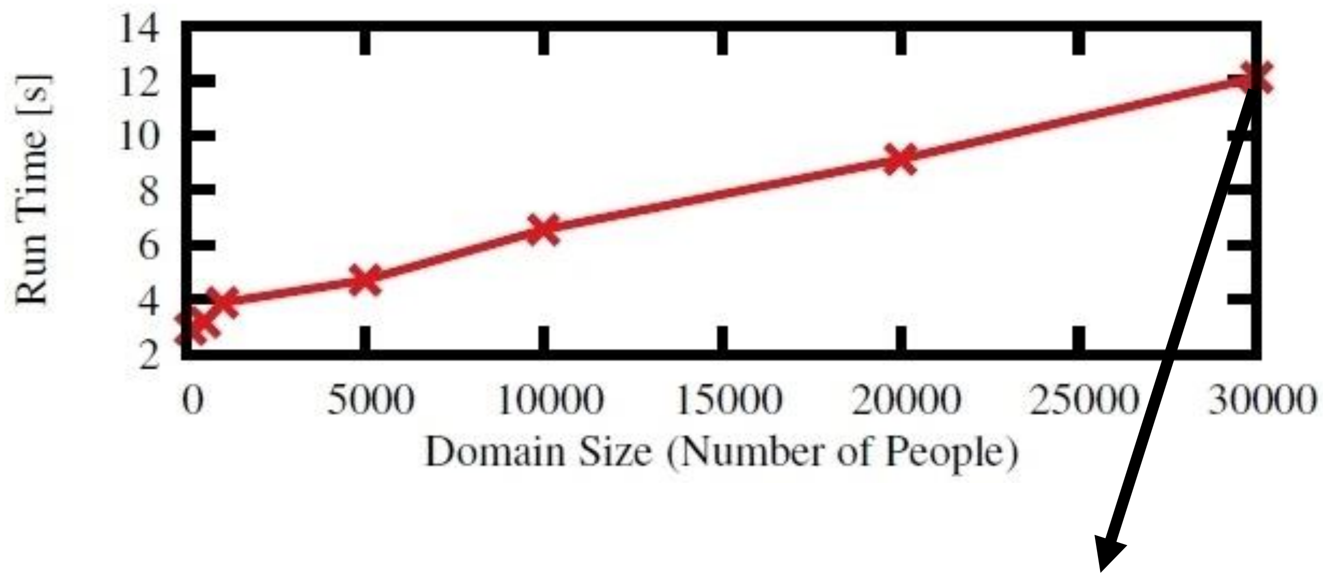
$$\mathbb{E}_w[n_F] = \Pr(F\theta_1) + \cdots + \Pr(F\theta_m)$$

- **Idea:** Lift the computation of $\mathbb{E}_w[n_j]$

[Van Haaren'16]

# Learning Time

w Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)



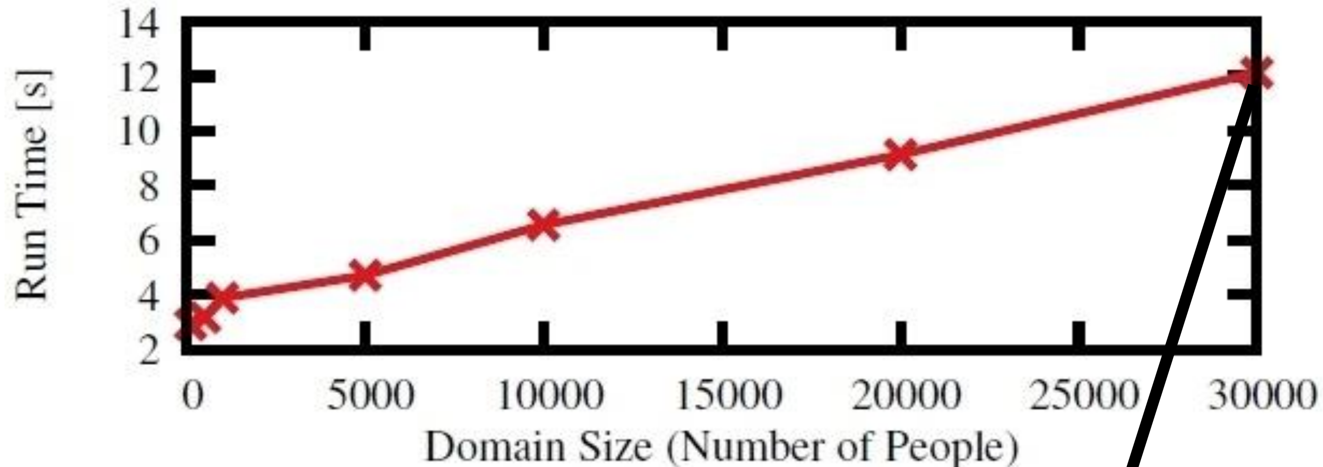

Big data

Learns a model over 900,030,000 random variables

[Van Haaren'16]

# Learning Time

w Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)




~~Big data~~

Big models

Learns a model over
900,030,000 random variables

[Van Haaren'16]

# More Lifted Algorithms

- Exact Inference (AI)
  - First-Order Variable Elimination [Poole’03, deSalvoBraz’05, Milch’08, Taghipour’13]
  - First-Order Knowledge Compilation [V.d.Broeck’11,’12,’13]
  - Probabilistic Theorem Proving [Gogate’11]
  - MPE/MAP Inference [deSalvoBraz’06,Apsel’12,Sarkhel’14,Kopp’15]

# More Lifted Algorithms

- Approximate Inference (AI)
  - Lifted Belief Propagation [Jaimovich’07, Singla’08, Kersting’09]
  - Lifted Bisimulation/Mini-buckets [Sen’08,‘09]
  - Lifted Importance Sampling [Gogate’11,’12]
  - Lifted Relax, Compensate & Recover [V.d.Broeck’12]
  - Lifted MCMC [Niepert’13,Venugopal’12,VdB’15]
  - Lifted Variational Inference [Choi’12, Bui’12]
  - Lifted MAP-LP [Mladenov’14, Apsel’14]

# More Lifted Algorithms

- Other Tasks (AI)
  - Lifted Kalman Filter [Ahmadi'11, Choi'11]
  - Lifted Linear Programming [Mladenov'12]
- Surveys [Kersting'12,Kimmig'15]
- Approximate Query Evaluation (DB)
  - Dissociation [Gatterbauer'13,'14,'15]
  - Collapsed Sampling [Gribkoff'15]
  - Approximate Compilation [Olteanu'10, Dylla'13]

# Conclusions

- A radically new reasoning paradigm
- Lifted inference is **frontier** and **integration** of AI, KR, ML, DBs, theory, etc.
- We need
  - relational databases and logic
  - probabilistic models and statistical learning
  - algorithms that scale
- Many theoretical open problems
- Recently cool practical applications

# Symmetric Open Problems

- Rules are complete beyond $FO^2$?
- Lifted approximations
  - Over-symmetric approx. with guarantees
  - Combined with Learning
- Mixed symmetric and asymmetric
- Theoretical computer science connections
  - Understanding #P1
- More SRL applications
- More expressive logics and programs
- Continuous random variables + Logic

# Asymmetric Open Problems

- Extensions of the Dichotomy theorem
  - For 0, ½, 1 probabilities
  - FDs, Deterministic tables
  - Negations: ∀FO, ∃FO, or full FO
- Lifted approximation algorithms
- Characterize queries with tractable compilation to: FBDD, SDD, d-DNNF
- Circuit language supporting dichotomy
- Characterize queries with tractable most likely world (MAP = maximum a posterior)

# Long-Term Outlook

Probabilistic inference and learning exploit

- ~ 1988: conditional independence
- ~ 2000: contextual independence (local structure)

# Long-Term Outlook

Probabilistic inference and learning exploit

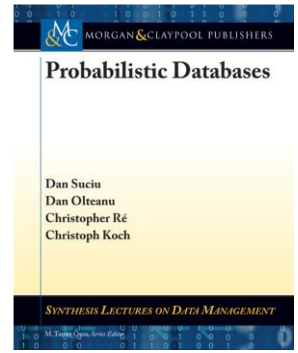
~ 1988: conditional independence

~ 2000: contextual independence (local structure)

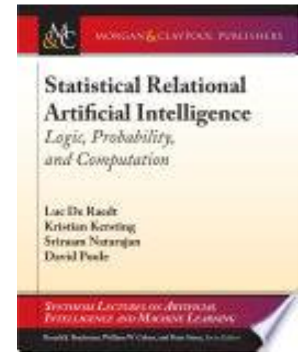
~ 201?: **symmetry & exchangeability & first-order**

# If you want more…

- Books
  - Probabilistic Databases
  - Statistical Relational AI
  - (Lifted Inference Book)




[Suciu'11]




[DeRaedt'16]

- StarAI workshop on Monday
  [http://www.starai.org](http://www.starai.org)

- Main conference papers

# Thank You!

Questions?

# References


- Suciu, Dan, Dan Olteanu, Christopher Ré, and Christoph Koch. "Probabilistic databases." Synthesis Lectures on Data Management 3, no. 2 (2011): 1-180.
- De Raedt, Luc, Kristian Kersting, Sriraam Natarajan, and David Poole. "Statistical Relational Artificial Intelligence: Logic, Probability, and Computation." Synthesis Lectures on Artificial Intelligence and Machine Learning 10, no. 2 (2016): 1-189.
- Gartner. "Market Share: Relational Database Management Systems by Operating System, Worldwide" (2006).
- Carlson, Andrew, Justin Betteridge, Bryan Kisiel, Burr Settles, Estevam R. Hruschka Jr, and Tom M. Mitchell. "Toward an Architecture for Never-Ending Language Learning." In AAAI, vol. 5, p. 3. 2010.
- Dong, Xin, Evgeniy Gabrilovich, Geremy Heitz, Wilko Horn, Ni Lao, Kevin Murphy, Thomas Strohmann, Shaohua Sun, and Wei Zhang. "Knowledge vault: A web-scale approach to probabilistic knowledge fusion." In Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining, pp. 601-610. ACM, 2014.

# References


- Niu, Feng, Ce Zhang, Christopher Ré, and Jude W. Shavlik. "DeepDive: Web-scale Knowledge-base Construction using Statistical Learning and Inference." VLDS 12 (2012): 25-28.
- Chen, Brian X. "Siri, Alexa and Other Virtual Assistants Put to the Test" The New York Times (2016).
- Richardson, Matthew, and Pedro Domingos. "Markov logic networks." Machine learning 62, no. 1-2 (2006): 107-136.
- Getoor, Lise, Ben Taskar. Introduction to statistical relational learning. MIT press, 2007.
- Van den Broeck, Guy, Ingo Thon, Martijn Van Otterlo, and Luc De Raedt. "DTProbLog: A decision-theoretic probabilistic Prolog." In Proceedings of the twenty-fourth AAAI conference on artificial intelligence, pp. 1217-1222. AAAI Press, 2010.
- Bach, Stephen, Bert Huang, Ben London, and Lise Getoor. "Hinge-loss Markov random fields: Convex inference for structured prediction." arXiv preprint arXiv:1309.6813 (2015).

# References


- Van den Broeck, Guy. "Towards high-level probabilistic reasoning with lifted inference." AAAI Spring Symposium on KRR (2015).
- Niepert, Mathias, and Guy Van den Broeck. "Tractability through exchangeability: A new perspective on efficient probabilistic inference." AAAI (2014).
- Van den Broeck, Guy. Lifted inference and learning in statistical relational models. Diss. Ph. D. Dissertation, KU Leuven, 2013.
- Vardi, Moshe Y. "The complexity of relational query languages." In Proceedings of the fourteenth annual ACM symposium on Theory of computing, pp. 137-146. ACM, 1982.
- Olteanu, Dan, and Jiewen Huang. "Using OBDDs for efficient query evaluation on probabilistic databases." In International Conference on Scalable Uncertainty Management, pp. 326-340. Springer Berlin Heidelberg, 2008.

# References


- Jha, Abhay, and Dan Suciu. "Knowledge compilation meets database theory: compiling queries to decision diagrams." ICDT (2011).
- Dalvi, Nilesh, and Dan Suciu. "The dichotomy of probabilistic inference for unions of conjunctive queries." Journal of the ACM (JACM) 59, no. 6 (2012): 30.
- Chavira, Mark, and Adnan Darwiche. "Compiling Bayesian networks with local structure." In IJCAI, vol. 5, pp. 1306-1312. 2005.
- Sang, Tian, Paul Beame, and Henry A. Kautz. "Performing Bayesian inference by weighted model counting." In AAAI, vol. 5, pp. 475-481. 2005.
- Chavira, Mark, and Adnan Darwiche. "On probabilistic inference by weighted model counting." Artificial Intelligence 172, no. 6 (2008): 772-799.

# References


- Choi, Arthur, Doga Kisa, and Adnan Darwiche. "Compiling probabilistic graphical models using sentential decision diagrams." In European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty, pp. 121-132. Springer Berlin Heidelberg, 2013.

- Chavira, Mark, Adnan Darwiche, and Manfred Jaeger. "Compiling relational Bayesian networks for exact inference." International Journal of Approximate Reasoning 42, no. 1 (2006): 4-20.

- Fierens, Daan, Guy Van den Broeck, Ingo Thon, Bernd Gutmann, and Luc De Raedt. "Inference in probabilistic logic programs using weighted CNF's." UAI (2011).

- Fierens, Daan, Guy Van den Broeck, Joris Renkens, Dimitar Shterionov, Bernd Gutmann, Ingo Thon, Gerda Janssens, and Luc De Raedt. "Inference and learning in probabilistic logic programs using weighted boolean formulas." Theory and Practice of Logic Programming 15, no. 03 (2015): 358-401.

- Van den Broeck, Guy, Nima Taghipour, Wannes Meert, Jesse Davis, and Luc De Raedt. "Lifted probabilistic inference by first-order knowledge compilation." In Proceedings of the Twenty-Second international joint conference on Artificial Intelligence, pp. 2178-2185. AAAI Press/International Joint Conferences on Artificial Intelligence, 2011.

# References


- Gogate, Vibhav, and Pedro Domingos. "Probabilistic theorem proving." UAI , 2011.
- Van den Broeck, Guy, Wannes Meert, and Adnan Darwiche. "Skolemization for weighted first-order model counting." In Proceedings of the 14th International Conference on Principles of Knowledge Representation and Reasoning (KR). 2014.
- Gribkoff, Eric, Guy Van den Broeck, and Dan Suciu. "Understanding the complexity of lifted inference and asymmetric weighted model counting." UAI, 2014.
- Jha, Abhay, and Dan Suciu. "Probabilistic databases with MarkoViews." Proceedings of the VLDB Endowment 5, no. 11 (2012): 1160-1171.
- Van den Broeck, Guy. "On the completeness of first-order knowledge compilation for lifted probabilistic inference." In Advances in Neural Information Processing Systems, pp. 1386-1394. 2011.
- Jaeger, Manfred, and Guy Van den Broeck. "Liftability of probabilistic inference: Upper and lower bounds." In Proceedings of the 2nd international workshop on statistical relational AI. 2012.

# References


- Darwiche, Adnan. "Decomposable negation normal form." Journal of the ACM (JACM) 48, no. 4 (2001): 608-647.
- Cook, Stephen A. "The complexity of theorem-proving procedures." In Proceedings of the third annual ACM symposium on Theory of computing, pp. 151-158. ACM, 1971.
- Valiant, Leslie G. "The complexity of computing the permanent." Theoretical computer science 8, no. 2 (1979): 189-201.
- Provan, J. Scott, and Michael O. Ball. "The complexity of counting cuts and of computing the probability that a graph is connected." SIAM Journal on Computing 12, no. 4 (1983): 777-788.
- Dalvi, Nilesh, and Dan Suciu. "Efficient query evaluation on probabilistic databases." The VLDB Journal 16, no. 4 (2004): 523-544.

# References


- Gomes, Carla P., Ashish Sabharwal, and Bart Selman. "Model counting." (2008).
- Huang, Jinbo, and Adnan Darwiche. "DPLL with a trace: From SAT to knowledge compilation." In IJCAI, vol. 5, pp. 156-162. 2005.
- Razgon, Igor. "On the read-once property of branching programs and CNFs of bounded treewidth." Algorithmica (2015): 1-18.
- Beame, Paul, Jerry Li, Sudeepa Roy, and Dan Suciu. "Model counting of query expressions: Limitations of propositional methods." ICDT (2014).
- Bova, Simone. "SDDs are Exponentially More Succinct than OBDDs." AAAI (2016).
- Beame, Paul, and Vincent Liew. "New Limits for Knowledge Compilation and Applications to Exact Model Counting." UAI (2015).

# References


- Valiant, Leslie G. "The complexity of enumeration and reliability problems." SIAM Journal on Computing 8, no. 3 (1979): 410-421.
- Bertoni, Alberto, Massimiliano Goldwurm, and Nicoletta Sabadini. "The complexity of computing the number of strings of given length in context-free languages." Theoretical Computer Science 86, no. 2 (1991): 325-342.
- Beame, Paul, Guy Van den Broeck, Eric Gribkoff, and Dan Suciu. "Symmetric weighted first-order model counting." In Proceedings of the 34th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems, pp. 313-328. ACM, 2015.
- Fagin, Ronald. "Generalized first-order spectra and polynomial-time recognizable sets." (1974).
- Ceylan, Ismail Ilkan, Adnan Darwiche, and Guy Van den Broeck. "Open-world probabilistic databases." Proc. of KR 16 (2016).

# References


- De Raedt, Luc, Angelika Kimmig, and Hannu Toivonen. "ProbLog: A Probabilistic Prolog and Its Application in Link Discovery." In IJCAI, vol. 7, pp. 2462-2467. 2007.

- Van den Broeck, Guy, and Adnan Darwiche. "On the complexity and approximation of binary evidence in lifted inference." In Advances in Neural Information Processing Systems, pp. 2868-2876. 2013.

- Van den Broeck, Guy, and Mathias Niepert. "Lifted probabilistic inference for asymmetric graphical models." AAAI (2015).

- Venugopal, Deepak, and Vibhav Gogate. "Evidence-based clustering for scalable inference in markov logic." In Joint European Conference on Machine Learning and Knowledge Discovery in Databases, pp. 258-273. Springer Berlin Heidelberg, 2014.

- Van Haaren, Jan, Guy Van den Broeck, Wannes Meert, and Jesse Davis. "Lifted generative learning of Markov logic networks." Machine Learning 103, no. 1 (2016): 27-55.

# References


- Poole, David. "First-order probabilistic inference." In IJCAI, vol. 3, pp. 985-991. 2003.
- de Salvo Braz, Rodrigo, Eyal Amir, and Dan Roth. "Lifted first-order probabilistic inference." In Proceedings of the 19th international joint conference on Artificial intelligence, pp. 1319-1325. 2005.
- Milch, Brian, Luke S. Zettlemoyer, Kristian Kersting, Michael Haimes, and Leslie Pack Kaelbling. "Lifted Probabilistic Inference with Counting Formulas." In Aaai, vol. 8, pp. 1062-1068. 2008.
- Taghipour, Nima, Daan Fierens, Jesse Davis, and Hendrik Blockeel. "Lifted Variable Elimination with Arbitrary Constraints." In AISTATS, pp. 1194-1202. 2012.
- Van den Broeck, Guy, and Jesse Davis. "Conditioning in first-order knowledge compilation and lifted probabilistic inference." In Proceedings of the Twenty-Sixth AAAI Conference on Artificial Intelligence, pp. 1-7. AAAI Press, 2012.

# References


- de Salvo Braz, Rodrigo, Eyal Amir, and Dan Roth. "MPE and partial inversion in lifted probabilistic variable elimination." In AAAI, vol. 6, pp. 1123-1130. 2006.

- Apsel, Udi, and Ronen I. Brafman. "Exploiting uniform assignments in first-order mpe." UAI (2012).

- Sarkhel, Somdeb, Deepak Venugopal, Parag Singla, and Vibhav Gogate. "Lifted MAP Inference for Markov Logic Networks." In AISTATS, pp. 859-867. 2014.

- Kopp, Timothy, Parag Singla, and Henry Kautz. "Lifted symmetry detection and breaking for map inference." In Advances in Neural Information Processing Systems, pp. 1315-1323. 2015.

- Jaimovich, Ariel, Ofer Meshi, and Nir Friedman. "Template based inference in symmetric relational Markov random fields." UAI (2007).

- Singla, Parag, and Pedro M. Domingos. "Lifted First-Order Belief Propagation." In AAAI, vol. 8, pp. 1094-1099. 2008.

# References


- Kersting, Kristian, Babak Ahmadi, and Sriraam Natarajan. "Counting belief propagation." In Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence, pp. 277-284. AUAI Press, 2009.
- Sen, Prithviraj, Amol Deshpande, and Lise Getoor. "Exploiting shared correlations in probabilistic databases." Proceedings of the VLDB Endowment 1, no. 1 (2008): 809-820.
- Sen, Prithviraj, Amol Deshpande, and Lise Getoor. "Bisimulation-based approximate lifted inference." In Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence, pp. 496-505. AUAI Press, 2009.
- Gogate, Vibhav, Abhay Kumar Jha, and Deepak Venugopal. "Advances in Lifted Importance Sampling." In AAAI. 2012.
- Van den Broeck, Guy, Arthur Choi, and Adnan Darwiche. "Lifted relax, compensate and then recover: From approximate to exact lifted probabilistic inference." UAI (2012).

# References


- Niepert, Mathias. "Symmetry-aware marginal density estimation." AAAI (2013).
- Venugopal, Deepak, and Vibhav Gogate. "On lifting the gibbs sampling algorithm." In Advances in Neural Information Processing Systems, pp. 1655-1663. 2012.
- Choi, Jaesik, and Eyal Amir. "Lifted relational variational inference." UAI (2012).
- Bui, Hung Hai, Tuyen N. Huynh, and Sebastian Riedel. "Automorphism groups of graphical models and lifted variational inference." StarAI (2012).
- Mladenov, Martin, Kristian Kersting, and Amir Globerson. "Efficient Lifting of MAP LP Relaxations Using k-Locality." In AISTATS, pp. 623-632. 2014.
- Apsel, Udi, Kristian Kersting, and Martin Mladenov. "Lifting Relational MAP-LPs Using Cluster Signatures." In AAAI Workshop: Statistical Relational Artificial Intelligence. 2014.

# References


- Ahmadi, Babak, Kristian Kersting, and Scott Sanner. "Multi-evidence lifted message passing, with application to pagerank and the kalman filter." In IJCAI Proceedings-International Joint Conference on Artificial Intelligence, vol. 22, no. 1, p. 1152. 2011.
- Choi, Jaesik, Abner Guzman-Rivera, and Eyal Amir. "Lifted Relational Kalman Filtering." In IJCAI, pp. 2092-2099. 2011.
- Mladenov, Martin, Babak Ahmadi, and Kristian Kersting. "Lifted Linear Programming." In AISTATS, pp. 788-797. 2012.
- Kimmig, Angelika, Lilyana Mihalkova, and Lise Getoor. "Lifted graphical models: a survey." Machine Learning 99, no. 1 (2015): 1-45.
- Kersting, Kristian. "Lifted Probabilistic Inference." In ECAI, pp. 33-38. 2012.
- Gatterbauer, Wolfgang, and Dan Suciu. "Dissociation and propagation for efficient query evaluation over probabilistic databases." arXiv preprint arXiv:1310.6257 (2013).

# References


- Gatterbauer, Wolfgang, and Dan Suciu. "Oblivious bounds on the probability of boolean functions." ACM Transactions on Database Systems (TODS) 39, no. 1 (2014): 5.
- Gatterbauer, Wolfgang, and Dan Suciu. "Approximate lifted inference with probabilistic databases." Proceedings of the VLDB Endowment 8, no. 5 (2015): 629-640.
- Gribkoff, Eric, and Dan Suciu. "SlimShot: Probabilistic inference for web-scale knowledge bases." (2015).
- Olteanu, Dan, Jiewen Huang, and Christoph Koch. "Approximate confidence computation in probabilistic databases." In 2010 IEEE 26th International Conference on Data Engineering (ICDE 2010), pp. 145-156. IEEE, 2010.
- Dylla, Maximilian, Iris Miliaraki, and Martin Theobald. "Top-k query processing in probabilistic databases with non-materialized views." In Data Engineering (ICDE), 2013 IEEE 29th International Conference on, pp. 122-133. IEEE, 2013.